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Generalized ulam-hyers stability of C^* -Ternary algebra n -Homomorphisms for a functional equation

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Abstract

In this article, we investigate the Ulam-Hyers stability of C^* -ternary algebra n -homomorphisms for the functional equation:

$$f(x_{1,1} + x_{2,1}, \dots, x_{1,n} + x_{2,n}) = \sum_{1 \leq i_1, \dots, i_n \leq 2} f(x_{i_1,1}, \dots, x_{i_n,n})$$

in C^* -ternary algebras.

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1. Introduction and preliminaries

Ternary algebraic operations were considered in the nineteenth century by several mathematicians, such as Cayley [1] who introduced the notion of cubic matrix, which, in turn, was generalized by Kapranov et al. [2]. The simplest example of such nontrivial ternary operation is given by the following composition rule:

$$\{a, b, c\}_{ijk} = \sum_{1 \leq l, m, n \leq N} a_{nil} b_{ljm} c_{mkn} \quad (i, j, k = 1, 2, \dots, N).$$

Ternary structures and their generalization, the so-called n -array structures, raise certain hopes in view of their applications in physics. Some significant physical applications are as follows (see [3]):

- (1) The algebra of *nonions* generated by two matrices

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \& \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega \\ \omega^2 & 0 & 0 \end{pmatrix} \quad \left(\omega = e^{\frac{2\pi i}{3}} \right)$$

was introduced by Sylvester as a ternary analog of Hamilton's quaternions [4].

- (2) The quark model inspired a particular brand of ternary algebraic systems. The so-called *Nambu mechanics* is based on such structures [5].

There are also some applications, although still hypothetical, in the fractional quantum Hall effect, the non-standard statistics, supersymmetric theory, and Yang-Baxter equation [4,6].

A C^* -ternary algebra is a complex Banach space A , equipped with a ternary product $(x, y, z) \mapsto [x, y, z]$ of A^3 into A , which is \mathbb{C} -linear in the outer variables, conjugate \mathbb{C} -linear in the middle variable, and associative in the sense that $[x, y, [z, w, v]] = [x, [w, z, y], v] = [[x, y, z], w, v]$, and satisfies $\|[x, y, z]\| \leq \|x\| \cdot \|y\| \cdot \|z\|$ and $\|[x, x, x]\| = \|x\|^3$ (see [7,8]). Every left Hilbert C^* -module is a C^* -ternary algebra via the ternary product $[x, y, z] := \langle x, y \rangle z$.

If a C^* -ternary algebra $(A, [\cdot, \cdot, \cdot])$ has an identity, i.e., an element $e \in A$ such that $x = [x, e, e] = [e, e, x]$ for all $x \in A$, then it is customary to verify that A , endowed with $x \circ y := [x, e, y]$ and $x^* := [e, x, e]$, is a unital C^* -algebra. Conversely, if (A, \circ) is a unital C^* -algebra, then $[x, y, z] := x \circ y^* \circ z$ makes A into a C^* -ternary algebra.

Let A and B be C^* -ternary algebras. A \mathbb{C} -linear mapping $H : A \rightarrow B$ is called a C^* -ternary algebra homomorphism if

$$H([x, y, z]) = [H(x), H(y), H(z)]$$

for all $x, y, z \in A$.

Definition. Let A and B be C^* -ternary algebras. A multilinear mapping $H : A^n \rightarrow B$ over \mathbb{C} is called a C^* -ternary algebra n -homomorphism if it satisfies

$$H([x_1, y_1, z_1], \dots, [x_n, y_n, z_n]) = [H(x_1, \dots, x_n), \dots, H(z_1, \dots, z_n)]$$

for all $x_1, y_1, z_1, \dots, x_n, y_n, z_n \in A$.

In 1940, Ulam [9] gave a talk before the Mathematics Club of the University of Wisconsin in which he discussed a number of unsolved problems. Among these was the following question concerning the stability of homomorphisms:

We are given a group G and a metric group G' with metric $\rho(\cdot, \cdot)$. Given $\varepsilon > 0$, does there exist a $\delta > 0$ such that if $f : G \rightarrow G'$ satisfies $\rho(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G$, then a homomorphism $h : G \rightarrow G'$ exists with $\rho(f(x), h(x)) < \varepsilon$ for all $x \in G$?

In 1941, Hyers [10] gave the first partial solution to Ulam's question for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. Then, Aoki [11] and Bourgin [12] considered the stability problem with unbounded Cauchy differences. In 1978, Rassias [13] generalized the theorem of Hyers [10] by considering the stability problem with unbounded Cauchy differences. In 1991, Gajda [14], following the same approach as that by Rassias [13], gave an affirmative solution to this question for $p > 1$. It was shown by Gajda [14] as well as by Rassias and Šemrl [15], that one cannot prove a Rassias-type theorem when $p = 1$. Găvruta [16] obtained the generalized result of Rassias's theorem which allows the Cauchy difference to be controlled by a general unbounded function. During the last two decades, a number of articles and research monographs have been published on various generalizations and applications of the generalized Hyers-Ulam stability to a number of functional equations and mappings, for example, Cauchy-Jensen mappings, k -additive mappings, invariant means, multiplicative mappings, bounded n th differences, convex functions, generalized orthogonality mappings, Euler-Lagrange functional equations, differential equations, and Navier-Stokes equations. The instability of characteristic flows of solutions of partial differential equations is related to the Ulam's stability of functional equations [17]-[24]. On the other hand, the authors [25], Park [20] and Kim [26] have contributed studies in respect of the stability problem of ternary homomorphisms and ternary derivations.

2. Solution and stability

Let X and Y be real or complex vector spaces and $n \geq 2$ an integer. For a mapping $f: X^n \rightarrow Y$, consider the functional equation:

$$\begin{aligned} & f(x_{1,1} + x_{2,1}, \dots, x_{1,n} + x_{2,n}) \\ &= \sum_{1 \leq i_1, \dots, i_n \leq 2} f(x_{i_1,1}, \dots, x_{i_n,n}). \end{aligned} \quad (2.1)$$

The above functional equation is rewritten as

$$f(x_1 + \gamma_1, \dots, x_n + \gamma_n) = \sum_{(z_1, \dots, z_n) \in V} f(z_1, \dots, z_n), \quad (2.2)$$

where

$$\begin{aligned} V &= V(x_1, \gamma_1, \dots, x_n, \gamma_n) \\ &:= \{ (z_1, \dots, z_n) \mid z_1 \in \{x_1, \gamma_1\}, \dots, z_n \in \{x_n, \gamma_n\} \}. \end{aligned}$$

We solve the general problem in vector spaces for the n -additive mappings satisfying (2.1).

Theorem 2.1. *A mapping $f: X^n \rightarrow Y$ satisfies the equation (2.1) if and only if the mapping f is n -additive.*

Proof. Assume that f satisfies (2.1). Letting $x_1 = \gamma_1 = \dots = x_n = \gamma_n = 0$ in (2.2), we get $f(0, \dots, 0) = 0$. Letting $\gamma_1 = x_2 = \gamma_2 = \dots = x_n = \gamma_n = 0$ in (2.2), we have

$$f(x_1, 0, \dots, 0) = 0$$

for all $x_1 \in X$. Similarly, we get

$$f(0, x_2, 0, \dots, 0) = \dots = f(0, \dots, 0, x_n) = 0$$

for all $x_2, \dots, x_n \in X$. Setting $\gamma_1 = \gamma_2 = 0$ and $x_3 = \gamma_3 = \dots = x_n = \gamma_n = 0$ in (2.2), we have

$$f(x_1, x_2, 0, \dots, 0) = 0$$

for all $x_1, x_2 \in X$. Similarly, we get $f(x_1, 0, x_3, 0, \dots, 0) = \dots = f(0, \dots, 0, x_{n-1}, x_n) = 0$ for all $x_1, \dots, x_n \in X$.

Continuing this process, we obtain that $f(x_1, \dots, x_n) = 0$ for all $x_1, \dots, x_n \in X$ with $x_i = 0$ for some $i = 1, \dots, n$. Letting $\gamma_2 = \dots = \gamma_n = 0$ in (2.2), we get the additivity in the first variable. Similarly, the additivities in the remaining variables hold.

The converse is obvious. \square

We investigate the generalized Ulam's stability in C^* -ternary algebras for the n -additive mappings satisfying (2.1).

Lemma 2.2. *Let X and Y be complex vector spaces and let $f: X^n \rightarrow Y$ be a n -additive mapping such that*

$$f(\lambda_1 x_1, \dots, \lambda_n x_n) = \lambda_1 \dots \lambda_n f(x_1, \dots, x_n)$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ and all $x_1, \dots, x_n \in X$, then f is n -linear over \mathbb{C} .

Proof. Since f is n -additive, we get $f(\frac{1}{2}x_1, \dots, \frac{1}{2}x_n) = \frac{1}{2^n}f(x_1, \dots, x_n)$ for all $x_1, \dots, x_n \in X$. Now let $\sigma_1, \dots, \sigma_n \in \mathbb{C}$ and M be an integer greater than $2(|\sigma_1| + \dots + |\sigma_n|)$.

Since $|\frac{\sigma_1}{M}|, \dots, |\frac{\sigma_n}{M}| < \frac{1}{2}$, there is $s_1, \dots, s_n \in (\frac{\pi}{3}, \frac{\pi}{2}]$ such that $|\frac{\sigma_1}{M}| = \cos s_1 = \frac{e^{is_1} + e^{-is_1}}{2}, \dots, |\frac{\sigma_n}{M}| = \cos s_n = \frac{e^{is_n} + e^{-is_n}}{2}$. Now

$$\frac{\sigma_1}{M} = \left| \frac{\sigma_1}{M} \right| \lambda_1, \dots, \frac{\sigma_n}{M} = \left| \frac{\sigma_n}{M} \right| \lambda_n$$

for some $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$. Thus, we have

$$\begin{aligned} f(\sigma_1 x_1, \dots, \sigma_n x_n) &= f\left(M \frac{\sigma_1}{M} x_1, \dots, M \frac{\sigma_n}{M} x_n\right) \\ &= M^n f\left(\frac{\sigma_1}{M} x_1, \dots, \frac{\sigma_n}{M} x_n\right) = M^n f\left(\left| \frac{\sigma_1}{M} \right| \lambda_1 x_1, \dots, \left| \frac{\sigma_n}{M} \right| \lambda_n x_n\right) \\ &= M^n f\left(\frac{e^{is_1} + e^{-is_1}}{2} \lambda_1 x_1, \dots, \frac{e^{is_n} + e^{-is_n}}{2} \lambda_n x_n\right) \\ &= \frac{1}{2^n} M^n f(e^{is_1} \lambda_1 x_1 + e^{-is_1} \lambda_1 x_1, \dots, e^{is_n} \lambda_n x_n + e^{-is_n} \lambda_n x_n) \\ &= \frac{1}{2^n} M^n \sum_{r_1, \dots, r_n=1, -1} e^{r_1 is_1} \dots e^{r_n is_n} \lambda_1 \dots \lambda_n f(x_1, \dots, x_n) \\ &= \sigma_1 \dots \sigma_n f(x_1, \dots, x_n) \end{aligned}$$

for all $x_1, \dots, x_n \in X$. Hence, the mapping $f: X^n \rightarrow Y$ is n -linear over \mathbb{C} . \square

Using the above lemma, one can obtain the following result.

Theorem 2.3. *Let X and Y be complex vector spaces and let $f: X^n \rightarrow Y$ be a mapping such that*

$$\begin{aligned} &f(\lambda_1 x_{1,1} + \lambda_1 x_{2,1}, \dots, \lambda_n x_{1,n} + \lambda_n x_{2,n}) \\ &= \lambda_1 \dots \lambda_n \sum_{1 \leq i_1, \dots, i_n \leq 2} f(x_{i_1,1}, \dots, x_{i_n,n}) \end{aligned} \quad (2.3)$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_{1,1}, x_{2,1}, \dots, x_{1,n}, x_{2,n} \in X$. Then, f is n -linear over \mathbb{C} .

Proof. Letting $\lambda_1 = \dots = \lambda_n = 1$, by Theorem 1.1, f is n -additive. Letting $x_{2,1} = \dots = x_{2,n} = 0$ in (2.3), we get $f(\lambda_1 x_1, \dots, \lambda_n x_n) = \lambda_1 \dots \lambda_n f(x_1, \dots, x_n)$ for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \dots, x_n \in X$. Hence, by Lemma 2.2, the mapping f is n -linear over \mathbb{C} . \square

From now on, assume that A is a C^* -ternary algebra with norm $\|\cdot\|_A$ and that B is a C^* -ternary algebra with norm $\|\cdot\|_B$.

For a given mapping $f: A^n \rightarrow B$, we define

$$\begin{aligned} &D_{\lambda_1, \dots, \lambda_n} f(x_{1,1}, x_{2,1}, \dots, x_{1,n}, x_{2,n}) \\ &:= f(\lambda_1 x_{1,1} + \lambda_1 x_{2,1}, \dots, \lambda_n x_{1,n} + \lambda_n x_{2,n}) \\ &\quad - \lambda_1 \dots \lambda_n \sum_{1 \leq i_1, \dots, i_n \leq 2} f(x_{i_1,1}, \dots, x_{i_n,n}) \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_{1,1}, x_{2,1}, \dots, x_{1,n}, x_{2,n} \in A$.

We prove the generalized Ulam-Hyers stability of homomorphisms in C^* -ternary algebras for the functional equation

$$D_{\lambda_1, \dots, \lambda_n} f(x_{1,1}, x_{2,1}, \dots, x_{1,n}, x_{2,n}) = 0.$$

Theorem 2.4. Let $p_1, \dots, p_n \in (0, \infty)$ with $\sum_{i=1}^n p_i < n$ and $\theta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping such that

$$\begin{aligned} & \|D_{\lambda_1, \dots, \lambda_n} f(x_1, y_1, \dots, x_n, y_n)\|_B \\ & \leq \theta \prod_{i=1}^n \max\{\|x_i\|_A, \|y_i\|_A\}^{p_i} \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} & \|f([x_1, y_1, z_1], \dots, [x_n, y_n, z_n]) \\ & - [f(x_1, \dots, x_n), f(y_1, \dots, y_n), f(z_1, \dots, z_n)]\|_B \\ & \leq \theta (\|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} + \|y_1\|_A^{p_1} \cdots \|y_n\|_A^{p_n} + \|z_1\|_A^{p_1} \cdots \|z_n\|_A^{p_n}) \end{aligned} \quad (2.5)$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}$ and all $x_1, y_1, z_1, \dots, x_n, y_n, z_n \in A$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H: A^n \rightarrow B$ such that

$$\begin{aligned} & \|f(x_1, \dots, x_n) - H(x_1, \dots, x_n)\|_B \\ & \leq \frac{\theta}{2^n - 2 \sum_{i=1}^n p_i} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned} \quad (2.6)$$

for all $x_1, \dots, x_n \in A$.

Proof. Letting $\lambda_1 = \dots = \lambda_n = 1$, $y_1 = x_1, \dots, y_n = x_n$ in (2.4), we gain

$$\begin{aligned} & \|f(2x_1, \dots, 2x_n) - 2^n f(x_1, \dots, x_n)\|_B \\ & \leq \theta \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned} \quad (2.7)$$

for all $x_1, \dots, x_n \in A$. Thus, we have

$$\begin{aligned} & \left\| \frac{1}{2^{n(j+1)}} f(2^{j+1}x_1, \dots, 2^{j+1}x_n) - \frac{1}{2^{nj}} f(2^jx_1, \dots, 2^jx_n) \right\|_B \\ & \leq 2^{(\sum_{i=1}^n p_i - n)j - n} \theta \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned}$$

for all $x_1, \dots, x_n \in A$ and all $j \in \mathbb{N}$. For given integer l, m ($0 \leq l < m$), we obtain that

$$\begin{aligned} & \left\| \frac{1}{2^{nm}} f(2^m x_1, \dots, 2^m x_n) - \frac{1}{2^{nl}} f(2^l x_1, \dots, 2^l x_n) \right\|_B \\ & \leq \sum_{j=l}^{m-1} 2^{(\sum_{i=1}^n p_i - n)j - n} \theta \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned} \quad (2.8)$$

for all $x_1, \dots, x_n \in A$. Since $\sum_{i=1}^n p_i < n$, the sequence

$$\left\{ \frac{1}{2^{nj}} f(2^j x_1, \dots, 2^j x_n) \right\}$$

is a Cauchy sequence for all $x_1, \dots, x_n \in A$. Since B is complete, the sequence $\left\{ \frac{1}{2^{nj}} f(2^j x_1, \dots, 2^j x_n) \right\}$ converges for all $x_1, \dots, x_n \in A$. Define $H: A^n \rightarrow B$ by

$$H(x_1, \dots, x_n) := \lim_{j \rightarrow \infty} \frac{1}{2^{nj}} f(2^j x_1, \dots, 2^j x_n)$$

for all $x_1, \dots, x_n \in A$. Letting $l = 0$ and taking $m \rightarrow \infty$ in (2.8), one can obtain the inequality (2.6). By (2.4), we see that

$$\begin{aligned} & \left\| \frac{1}{2^{ns}} f(2^s(x_1 + y_1), \dots, 2^s(x_n + y_n)) \right. \\ & \quad \left. - \sum_{(z_1, \dots, z_n) \in V} \frac{1}{2^{ns}} f(2^s z_1, \dots, 2^s z_n) \right\|_B \\ & \leq 2^{(\sum_{i=1}^n p_i - n)s} \theta \prod_{i=1}^n \max\{\|x_i\|_A, \|y_i\|_A\}^{p_i} \end{aligned}$$

for all $x_1, y_1, \dots, x_n, y_n \in A$ and all s . Since $\sum_{i=1}^n p_i < n$, letting $s \rightarrow \infty$ in the above inequality, H satisfies (2.1). By Theorem 2.1, H is n -additive.

Letting $y_1 = x_1, \dots, y_n = x_n$ in (2.4), we gain

$$\begin{aligned} & \|f(2\lambda_1 x_1, \dots, 2\lambda_n x_n) - 2^n \lambda_1 \dots \lambda_n f(x_1, \dots, x_n)\|_B \\ & \leq \theta \|x_1\|_A^{p_1} \dots \|x_n\|_A^{p_n} \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \dots, x_n \in A$. Thus we have

$$\begin{aligned} & \|f(2^m \lambda_1 x_1, \dots, 2^m \lambda_n x_n) - 2^n \lambda_1 \dots \lambda_n f(2^{m-1} x_1, \dots, 2^{m-1} x_n)\|_B \\ & \leq 2^{(m-1)\sum_{i=1}^n p_i} \theta \|x_1\|_A^{p_1} \dots \|x_n\|_A^{p_n} \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$, all $x_1, \dots, x_n \in A$ and all $m \in \mathbb{N}$. Hence, we get

$$\begin{aligned} & \|f(2^m x_1, \dots, 2^m x_n) - 2^n f(2^{m-1} x_1, \dots, 2^{m-1} x_n)\|_B \\ & \leq 2^{(m-1)\sum_{i=1}^n p_i} \theta \|x_1\|_A^{p_1} \dots \|x_n\|_A^{p_n} \end{aligned}$$

for all $x_1, \dots, x_n \in A$ and all $m \in \mathbb{N}$, and one can show that

$$\begin{aligned} & \|\lambda_1 \dots \lambda_n f(2^m x_1, \dots, 2^m x_n) \\ & \quad - 2^n \lambda_1 \dots \lambda_n f(2^{m-1} x_1, \dots, 2^{m-1} x_n)\|_B \\ & = \|\lambda_1 \dots \lambda_n \cdot \|f(2^m x_1, \dots, 2^m x_n) - 2^n f(2^{m-1} x_1, \dots, 2^{m-1} x_n)\|_B \\ & \leq 2^{(m-1)\sum_{i=1}^n p_i} \theta \|x_1\|_A^{p_1} \dots \|x_n\|_A^{p_n} \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$, all $x_1, \dots, x_n \in A$ and all $m \in \mathbb{N}$. Hence,

$$\begin{aligned} & \|f(2^m \lambda_1 x_1, \dots, 2^m \lambda_n x_n) - \lambda_1 \dots \lambda_n f(2^m x_1, \dots, 2^m x_n)\|_B \\ & \leq \|f(2^m \lambda_1 x_1, \dots, 2^m \lambda_n x_n) - 2^n \lambda_1 \dots \lambda_n f(2^{m-1} x_1, \dots, 2^{m-1} x_n)\|_B \\ & \quad + \|2^n \lambda_1 \dots \lambda_n f(2^{m-1} x_1, \dots, 2^{m-1} x_n) \\ & \quad - \lambda_1 \dots \lambda_n f(2^m x_1, \dots, 2^m x_n)\|_B \\ & \leq 2^{1+(m-1)\sum_{i=1}^n p_i} \theta \|x_1\|_A^{p_1} \dots \|x_n\|_A^{p_n} \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$, all $x_1, \dots, x_n \in A$ and all $m \in \mathbb{N}$. Since $\sum_{i=1}^n p_i < n$, we have

$$\frac{1}{2^{nm}} \|f(2^m \lambda_1 x_1, \dots, 2^m \lambda_n x_n) - \lambda_1 \dots \lambda_n f(2^m x_1, \dots, 2^m x_n)\|_B \rightarrow 0$$

as $m \rightarrow \infty$ for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \dots, x_n \in A$. Hence

$$\begin{aligned} & H(\lambda_1 x_1, \dots, \lambda_n x_n) = \lim_{m \rightarrow \infty} \frac{f(2^m \lambda_1 x_1, \dots, 2^m \lambda_n x_n)}{2^{nm}} \\ & = \lim_{m \rightarrow \infty} \lambda_1 \dots \lambda_n \frac{f(2^m x_1, \dots, 2^m x_n)}{2^{nm}} \\ & = \lambda_1 \dots \lambda_n H(x_1, \dots, x_n) \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \dots, x_n \in A$. From Lemma 2.2, the mapping $H : A^n \rightarrow B$ is n -linear over \mathbb{C} . It follows from (2.5) that

$$\begin{aligned} & \| H([x_1, \gamma_1, z_1], \dots, [x_n, \gamma_n, z_n]) \\ & \quad - [H(x_1, \dots, x_n), H(\gamma_1, \dots, \gamma_n), H(z_1, \dots, z_n)] \|_B \\ &= \lim_{m \rightarrow \infty} \frac{1}{2^{3nm}} \| f(2^{3m}[x_1, \gamma_1, z_1], \dots, 2^{3m}[x_n, \gamma_n, z_n]) \\ & \quad - [f(2^m x_1, \dots, 2^m x_n), f(2^m \gamma_1, \dots, 2^m \gamma_n), f(2^m z_1, \dots, 2^m z_n)] \|_B \\ &\leq \lim_{m \rightarrow \infty} \frac{\theta 2^{m \sum_{i=1}^n p_i}}{2^{3nm}} (\| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n} \\ & \quad + \| \gamma_1 \|_A^{p_1} \cdots \| \gamma_n \|_A^{p_n} + \| z_1 \|_A^{p_1} \cdots \| z_n \|_A^{p_n}) \\ &= 0 \end{aligned}$$

for all $x_1, \gamma_1, z_1, \dots, x_n, \gamma_n, z_n \in A$. So

$$\begin{aligned} & H([x_1, \gamma_1, z_1], \dots, [x_n, \gamma_n, z_n]) \\ &= [H(x_1, \dots, x_n), H(\gamma_1, \dots, \gamma_n), H(z_1, \dots, z_n)] \end{aligned}$$

for all $x_1, \gamma_1, z_1, \dots, x_n, \gamma_n, z_n \in A$.

Now, let $T : A^n \rightarrow B$ be another n -additive mapping satisfying (2.6). Then, we have

$$\begin{aligned} & \| H(x_1, \dots, x_n) - T(x_1, \dots, x_n) \|_B \\ &= \frac{1}{2^{nm}} \| H(2^m x_1, \dots, 2^m x_n) - T(2^m x_1, \dots, 2^m x_n) \|_B \\ &\leq \frac{1}{2^{nm}} \| H(2^m x_1, \dots, 2^m x_n) - f(2^m x_1, \dots, 2^m x_n) \|_B \\ & \quad + \frac{1}{2^{nm}} \| f(2^m x_1, \dots, 2^m x_n) - T(2^m x_1, \dots, 2^m x_n) \|_B \\ &\leq \frac{2(\sum_{i=1}^n p_i - n)m + 1}{2^n - 2^{\sum_{i=1}^n p_i}} \theta \| x_1 \|_A^{p_1} \cdots \| x_n \|_A^{p_n}, \end{aligned}$$

which tends to zero as $m \rightarrow \infty$ for all $x_1, \dots, x_n \in A$. Hence, we can conclude that $H(x_1, \dots, x_n) = T(x_1, \dots, x_n)$ for all $x_1, \dots, x_n \in A$. This proves the uniqueness of H .

Thus, the mapping $H : A \rightarrow B$ is a unique C^* -ternary algebra n -homomorphism satisfying (2.6). \square

Letting $p_1 = \dots = p_n = 0$ and $\theta = \varepsilon$ in Theorem 2.4, we obtain the Ulam-Hyers stability for the n -additive functional equation (2.1).

Corollary 2.5. *Let $\varepsilon \in (0, \infty)$ and let $f : A^n \rightarrow B$ be a mapping satisfying*

$$\| D_{\lambda_1, \dots, \lambda_n} f(x_1, \gamma_1, \dots, x_n, \gamma_n) \|_B \leq \varepsilon$$

and

$$\begin{aligned} & \| f([x_1, \gamma_1, z_1], \dots, [x_n, \gamma_n, z_n]) \\ & \quad - [f(x_1, \dots, x_n), f(\gamma_1, \dots, \gamma_n), f(z_1, \dots, z_n)] \|_B \leq 3\varepsilon \end{aligned}$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_1, \gamma_1, z_1, \dots, x_n, \gamma_n, z_n \in A$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H : A^n \rightarrow B$ such that

$$\| f(x_1, \dots, x_n) - H(x_1, \dots, x_n) \|_B \leq \frac{\varepsilon}{2^n - 1}$$

for all $x_1, \dots, x_n \in A$.

Example 2.6. We present the following counterexample modified by the well-known counterexample of Z. Gajda [14] for the functional equation (2.1). Set $\theta > 0$ and let

$$\mu := \frac{\theta}{6n2^n}.$$

Define a function $f: \mathbb{R}^n \rightarrow \mathbb{R}$ by

$$f(x_1, \dots, x_n) := \sum_{n=0}^{\infty} \frac{1}{2^n} \phi_{\mu}(2^n x_1, x_2, \dots, x_n)$$

for all $x_1, \dots, x_n \in \mathbb{R}$, where $\phi_{\mu}: \mathbb{R}^n \rightarrow \mathbb{R}$ is the function given by

$$\phi_{\mu}(x_1, \dots, x_n) := \begin{cases} \mu & \text{if } x_1 \cdots x_n \geq 1 \\ \mu x_1 \cdots x_n & \text{if } -1 < x_1 \cdots x_n < 1 \\ -\mu & \text{if } x_1 \cdots x_n \leq -1 \end{cases}$$

for all $x_1, \dots, x_n \in \mathbb{R}$. Define another function $g: \mathbb{R} \rightarrow \mathbb{R}$ by

$$g(x) := f(x, 1, \dots, 1) = \sum_{n=0}^{\infty} \frac{1}{2^n} \phi_{\mu}(2^n x, 1, \dots, 1)$$

for all $x \in \mathbb{R}$.

For all $m \in \mathbb{N}$ and all $x_1, \dots, x_{2^m} \in \mathbb{R}$, we assert that

$$\left| g\left(\sum_{i=1}^{2^m} x_i\right) - \sum_{i=1}^{2^m} g(x_i) \right| \leq \frac{m\theta}{n2^n} \sum_{i=1}^{2^m} |x_i|. \quad (2.9)$$

It was proved in [14] that

$$|g(x+y) - g(x) - g(y)| \leq \frac{\theta}{n2^n} (|x| + |y|)$$

for all $x, y \in \mathbb{R}$, that is, (2.9) holds for $m = 1$. For a fixed $k \in \mathbb{N}$, assume that (2.9) holds for $m = k$. Then, we have

$$\begin{aligned} & \left| g\left(\sum_{i=1}^{2^{k+1}} x_i\right) - \sum_{i=1}^{2^{k+1}} g(x_i) \right| \\ & \leq \left| g\left(\sum_{i=1}^{2^{k+1}} x_i\right) - g\left(\sum_{i=1}^{2^k} x_i\right) - g\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) \right| \\ & \quad + \left| g\left(\sum_{i=1}^{2^k} x_i\right) - g\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) - \sum_{i=1}^{2^{k+1}} g(x_i) \right| \\ & \leq \frac{\theta}{n2^n} \left(\left| \sum_{i=1}^{2^k} x_i \right| + \left| \sum_{i=2^k+1}^{2^{k+1}} x_i \right| \right) + g\left(\sum_{i=1}^{2^k} x_i\right) - \sum_{i=1}^{2^k} g(x_i) \\ & \quad + g\left(\sum_{i=2^k+1}^{2^{k+1}} x_i\right) - \sum_{i=2^k+1}^{2^{k+1}} g(x_i) \\ & \leq \frac{\theta}{n2^n} \left(\left| \sum_{i=1}^{2^k} x_i \right| + \left| \sum_{i=2^k+1}^{2^{k+1}} x_i \right| \right) + \frac{k\theta}{n2^n} \left(\sum_{i=1}^{2^k} |x_i| + \sum_{i=2^k+1}^{2^{k+1}} |x_i| \right) \\ & = \frac{(k+1)\theta}{n2^n} \sum_{i=1}^{2^{k+1}} |x_i| \end{aligned}$$

for all $x_1, \dots, x_{2^{k+1}} \in \mathbb{R}$, that is, (2.9) holds for $m = k + 1$. Hence, (2.9) holds for all $m \in \mathbb{R}$.

Note that

$$\begin{aligned} f(x_1, \dots, x_n) &= \begin{cases} \mu & \text{if } 2^n x_1 \cdots x_n \geq 1 \\ \mu 2^n x_1 \cdots x_n & \text{if } -1 < 2^n x_1 \cdots x_n < 1 \\ -\mu & \text{if } 2^n x_1 \cdots x_n \leq -1 \end{cases} \\ &= f(x_1 \cdots x_n, 1, \dots, 1) = g(x_1 \cdots x_n) \end{aligned} \quad (2.10)$$

for all $x_1, \dots, x_n \in \mathbb{R}$. By the inequality (2.9) and the above equality, we see that

$$\begin{aligned} & \left| f(x_1 + \gamma_1, \dots, x_n + \gamma_n) - \sum_{(z_1, \dots, z_n) \in V} f(z_1, \dots, z_n) \right| \\ &= \left| g((x_1 + \gamma_1) \cdots (x_n + \gamma_n)) - \sum_{(z_1, \dots, z_n) \in V} g(z_1 \cdots z_n) \right| \\ &= \left| g\left(\sum_{(z_1, \dots, z_n) \in V} z_1 \cdots z_n\right) - \sum_{(z_1, \dots, z_n) \in V} g(z_1 \cdots z_n) \right| \\ &\leq \frac{\theta}{2^n} \sum_{(z_1, \dots, z_n) \in V} |z_1 \cdots z_n| \leq \frac{\theta}{2^n} (|x_1| + |\gamma_1|) \cdots (|x_n| + |\gamma_n|) \\ &= \theta \prod_{i=1}^n \max\{|x_i|, |\gamma_i|\}^1 \end{aligned}$$

for all $x_1, \gamma_1, \dots, x_n, \gamma_n \in \mathbb{R}$. However, we observe from [14] that

$$\frac{g(x^n)}{x^n} \rightarrow \infty \quad \text{as } x \rightarrow \infty.$$

and so

$$\frac{f(x, \dots, x)}{x^n} \rightarrow \infty \quad \text{as } x \rightarrow \infty.$$

Thus,

$$\frac{|f(x, \dots, x) - h(x, \dots, x)|}{x^n} (x \neq 0) \text{ is unbounded,}$$

where $h: \mathbb{R}^n \rightarrow \mathbb{R}$ is the function given by

$$h(x_1, \dots, x_n) := \lim_{m \rightarrow \infty} \frac{1}{2^{mn}} f(2^m x_1, \dots, 2^m x_n)$$

for all $x_1, \dots, x_n \in \mathbb{R}$. Hence, the function f is a counterexample for the singular case $\sum_{i=1}^n p_i = \text{nof Theorem 2.4}$.

Theorem 2.7. Let $p \in (0, n)$ and $\theta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping such that

$$\begin{aligned} & \|D_{\lambda_1, \dots, \lambda_n} f(x_1, \gamma_1, \dots, x_n, \gamma_n)\|_B \\ &\leq \theta \sum_{i=1}^n (\|x_i\|_A^p + \|\gamma_i\|_A^p) \end{aligned} \quad (2.11)$$

and

$$\begin{aligned} & \| f([x_1, y_1, z_1], \dots, [x_n, y_n, z_n]) \\ & - [f(x_1, \dots, x_n), f(y_1, \dots, y_n), f(z_1, \dots, z_n)] \|_B \\ & \leq \theta \sum_{i=1}^n (\|x_i\|_A^p + \|y_i\|_A^p + \|z_i\|_A^p) \end{aligned} \quad (2.12)$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}$ and all $x_1, y_1, z_1, \dots, x_n, y_n, z_n \in A$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H : A^n \rightarrow B$ such that

$$\| f(x_1, \dots, x_n) - H(x_1, \dots, x_n) \|_B \leq \frac{2\theta}{2^n - 2^p} \sum_{i=1}^n \|x_i\|_A^p$$

for all $x_1, \dots, x_n \in A$.

Proof. The proof is similar to the proof of Theorem 2.4. \square

Example 2.8. We present the following counterexample modified by the well-known counterexample of Z. Gajda [14] for the functional equation (2.1). Set $\theta > 0$ and let

$$\mu := \frac{\theta}{6n2^n}$$

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ be the same as in Example 2.6. By the same argument as in Example 2.6, for all $m \in \mathbb{N}$ and all $x_1, \dots, x_{2^m} \in \mathbb{R}$, one can obtain that g satisfies the inequality:

$$\left| g\left(\sum_{i=1}^{2^m} x_i\right) - \sum_{i=1}^{2^m} g(x_i) \right| \leq \frac{m\theta}{n2^n} \sum_{i=1}^{2^m} |x_i|.$$

By the equality (2.10) and the above inequality, we see that

$$\begin{aligned} & \left| f(x_1 + y_1, \dots, x_n + y_n) - \sum_{(z_1, \dots, z_n) \in V} f(z_1, \dots, z_n) \right| \\ & = \left| g((x_1 + y_1) \cdots (x_n + y_n)) - \sum_{(z_1, \dots, z_n) \in V} g(z_1 \cdots z_n) \right| \\ & = \left| g\left(\sum_{(z_1, \dots, z_n) \in V} z_1 \cdots z_n\right) - \sum_{(z_1, \dots, z_n) \in V} g(z_1 \cdots z_n) \right| \\ & \leq \frac{\theta}{2^n} \sum_{(z_1, \dots, z_n) \in V} |z_1 \cdots z_n| \end{aligned}$$

for all $x_1, y_1, \dots, x_n, y_n \in \mathbb{R}$. For each $x_1, y_1, \dots, x_n, y_n \in \mathbb{R}$, let $M(x_1, y_1, \dots, x_n, y_n) := \max\{|x_1|, |y_1|, \dots, |x_n|, |y_n|\}$. We have

$$\frac{\theta}{2^n} \sum_{(z_1, \dots, z_n) \in V} |z_1 \cdots z_n| \leq \frac{\theta}{2^n} \sum_{(z_1, \dots, z_n) \in V} M^n = \theta M^n \leq \theta \sum_{i=1}^n (|x_i|^n + |y_i|^n)$$

for all $x_1, y_1, \dots, x_n, y_n \in \mathbb{R}$. Thus we have

$$\left| f(x_1 + y_1, \dots, x_n + y_n) - \sum_{(z_1, \dots, z_n) \in V} f(z_1, \dots, z_n) \right| \leq \theta \sum_{i=1}^n (|x_i|^n + |y_i|^n)$$

for all $x_1, y_1, \dots, x_n, y_n \in \mathbb{R}$. By the same reason as for Example 2.6, the function f is a counterexample for the singular case $p = n$ of Theorem 2.7.

Theorem 2.9. Let $p_1, \dots, p_n \in (0, \infty)$ with $\sum_{i=1}^n p_i < n$, $s \in (0, n)$ and $\theta, \eta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping such that

$$\begin{aligned} & \|D_{\lambda_1, \dots, \lambda_n} f(x_{1,1}, x_{2,1}, \dots, x_{1,n}, x_{2,n})\|_B \\ & \leq \theta \max\{\|x_{1,1}\|_A, \|x_{2,1}\|_A\}^{p_1} \cdots \max\{\|x_{1,n}\|_A, \|x_{2,n}\|_A\}^{p_n} \\ & \quad + \eta \sum_{i=1}^n (\|x_{1,i}\|_A^s + \|x_{2,i}\|_A^s) \end{aligned} \quad (2.13)$$

and

$$\begin{aligned} & \|f([x_{1,1}, x_{2,1}, x_{3,1}], \dots, [x_{1,n}, x_{2,n}, x_{3,n}]) \\ & \quad - [f(x_{1,1}, \dots, x_{1,n}), f(x_{2,1}, \dots, x_{2,n}), f(x_{3,1}, \dots, x_{3,n})]\|_B \\ & \leq \theta \sum_{i=1}^3 \|x_{i,1}\|_A^{p_1} \cdots \|x_{i,n}\|_A^{p_n} + \eta \sum_{i=1}^n (\|x_{1,i}\|_A^s + \|x_{2,i}\|_A^s + \|x_{3,i}\|_A^s) \end{aligned} \quad (2.14)$$

for all $\lambda_1, \dots, \lambda_n \in \mathbb{T}^1$ and all $x_{1,1}, x_{2,1}, x_{3,1}, \dots, x_{1,n}, x_{2,n}, x_{3,n} \in A$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H: A^n \rightarrow B$ such that

$$\begin{aligned} & \|f(x_1, \dots, x_n) - H(x_1, \dots, x_n)\|_B \\ & \leq \frac{\theta}{2^n - 2^{\sum_{i=1}^n p_i}} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} + \frac{2\eta}{2^n - 2^s} \sum_{i=1}^n \|x_i\|_A^s \end{aligned}$$

for all $x_1, \dots, x_n \in A$.

Proof. The proof is similar to the proof of Theorem 2.4. \square

Theorem 2.10. Let $p_1, \dots, p_n \in (0, \infty)$ with $\sum_{i=1}^n p_i > 3n$ and $\theta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping satisfying (2.4) (2.5). Then, there exists a unique C^* -ternary algebra n -homomorphism $H: A^n \rightarrow B$ such that

$$\begin{aligned} & \|f(x_1, \dots, x_n) - H(x_1, \dots, x_n)\|_B \\ & \leq \frac{\theta}{2^{\sum_{i=1}^n p_i} - 2^n} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned} \quad (2.15)$$

for all $x_1, \dots, x_n \in A$.

Proof. It follows from (2.7) that

$$\left\| f(x_1, \dots, x_n) - 2^n f\left(\frac{x_1}{2}, \dots, \frac{x_n}{2}\right) \right\|_B \leq \frac{\theta}{2^{\sum_{i=1}^n p_i}} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n}$$

for all $x_1, \dots, x_n \in A$. Hence,

$$\begin{aligned} & \left\| 2^{nl} f\left(\frac{x_1}{2^l}, \dots, \frac{x_n}{2^l}\right) - 2^{nm} f\left(\frac{x_1}{2^m}, \dots, \frac{x_n}{2^m}\right) \right\|_B \\ & \leq \sum_{j=l}^{m-1} \left\| 2^{nj} f\left(\frac{x_1}{2^j}, \dots, \frac{x_n}{2^j}\right) - 2^{n(j+1)} f\left(\frac{x_1}{2^{j+1}}, \dots, \frac{x_n}{2^{j+1}}\right) \right\|_B \\ & \leq \frac{\theta}{2^{\sum_{i=1}^n p_i}} \sum_{j=l}^{m-1} \frac{2^{nj}}{2^{j \sum_{i=1}^n p_i}} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} \end{aligned} \quad (2.16)$$

for all nonnegative integers m and l with $m > l$ and all $x_1, \dots, x_n \in A$. It follows from (2.16) that the sequence $\left\{2^{nm}f\left(\frac{x_1}{2^m}, \dots, \frac{x_n}{2^m}\right)\right\}$ is a Cauchy sequence for all $x_1, \dots, x_n \in A$. Since B is complete, the sequence $\left\{2^{nm}f\left(\frac{x_1}{2^m}, \dots, \frac{x_n}{2^m}\right)\right\}$ converges. Hence, one can define the mapping $H: A^n \rightarrow B$ by $H(x_1, \dots, x_n) := \lim_{m \rightarrow \infty} 2^{nm}f\left(\frac{x_1}{2^m}, \dots, \frac{x_n}{2^m}\right)$ for all $x_1, \dots, x_n \in A$. Moreover, letting $l = 0$ and passing the limit $m \rightarrow \infty$ in (2.16), we get (2.15).

The remainder of the proof is similar to the proof of Theorem 2.4. \square

Theorem 2.11. *Let $p \in (3n, \infty)$ and $\theta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping satisfying (2.11) (2.7), and $f(0, \dots, 0) = 0$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H: A^n \rightarrow B$ such that*

$$\|f(x_1, \dots, x_n) - H(x_1, \dots, x_n)\|_B \leq \frac{2\theta}{2^p - 2^n} \sum_{i=1}^n \|x_i\|_A^p$$

for all $x_1, \dots, x_n \in A$.

Proof. The proof is similar to that of Theorem 2.10. \square

Theorem 2.12. *Let $p_1, \dots, p_n \in (0, \infty)$ with $\sum_{i=1}^n p_i > 3n$, $s \in (n, \infty)$ and $\theta, \eta \in (0, \infty)$, and let $f: A^n \rightarrow B$ be a mapping such that (2.13), (2.14), and $f(0, \dots, 0) = 0$. Then, there exists a unique C^* -ternary algebra n -homomorphism $H: A^n \rightarrow B$ such that*

$$\begin{aligned} & \|f(x_1, \dots, x_n) - H(x_1, \dots, x_n)\|_B \\ & \leq \frac{\theta}{2^{\sum_{i=1}^n p_i} - 2^n} \|x_1\|_A^{p_1} \cdots \|x_n\|_A^{p_n} + \frac{2\eta}{2^s - 2^n} \sum_{i=1}^n \|x_i\|_A^s \end{aligned}$$

for all $x_1, \dots, x_n \in A$.

Proof. The proof is similar to that of Theorem 2.10. \square

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Authors' contributions

All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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